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Loosely Bound States near the Charm Threshold.. Charm Molecules

S. NUSSINOV[+*]
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
Institute of Advanced Study, Princeton, N.J. 08540

and

D. P. SIDHU[**]
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
Brookhaven National Laboratory, Upton, N.Y. 11973

ABSTRACT

We discuss the possibility of loosely bound states near the charm threshold in the charmed particle and anti-particle system such as $D\bar{D}$, $D^*\bar{D}^*$ etc. An explicit calculation shows that the S and P-wave states exist in the $D\bar{D}$ system with reasonable values for the parameters of the interaction. These bound states may be very close to threshold with large sizes (rms $\gtrsim$ 1 fm). Characteristics of these molecular-like states and their relationship with the quark model $c\bar{c}$ bound states are also discussed.

[+]On leave from Tel Aviv University, Ramat-Aviv, Tel Aviv.

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I. INTRODUCTION

A simple approximation for ρ or ω as a non-relativistic bound state of two pseudoscalar mesons is inadequate for the following reasons:

(i) recoil effects are very large;

(ii) The bound state (being the ^3S $q\bar{q}$ quark model counterpart of the ^1S pseudoscalars) has roughly the same spatial size as the constituents $\{$and indeed experimentally $\sigma_T(\rho N) \approx \sigma_T(\pi N)\ldots\}$. This means that the two bound mesons strongly fuse together as one would schematically describe by the quark diagram (Fig. 1) to such an extent that we have no longer a $\bar{q}q$ $\bar{q}q$ but rather a $\bar{q}q$ state, suggesting that many t channel exchanges acting coherently may be important as shown in Fig. 2, which in turn may be related to the composite nature of the constituents. This picture suggests the possibility of couplings to the excited states (Fig. 3) and also together with the strong overlap (Fig. 4) allows for the massive particle exchanges to be important in the sum shown in Fig. 2.

Clearly all this does not mean that ω, say, is not a bound state in the $\bar{K}K$ channel and that S matrix methods which would take into account some of the effects mentioned above will not lead to its accurate determination. It only means that the simple N.R. potential description in terms of meson constituents may be inadequate.

However, we can envision other situations, specifically we will be considering $\bar{D}D$ bound states where D is the recently discovered resonance at 1865 MeV which is assumed to be the charmed member of SU(4) pseudoscalars where bona-fide non-relativistic $\bar{q}q\ \bar{q}q$ bound states exist near threshold which are quite distinct from the quark model qq states since

(i) The recoil effects $|p_r|/M_D \propto \sqrt{E_B/M_D}$ (binding energy divided by meson mass) are negligible.

(ii) The size of the bound state $R = (\sqrt{M_D E_B})^{-1}$ is larger than μ^{-1}_{meson} (μ is the characteristic mass of the exchanged meson) so that the average momentum carried by the virtual light meson exchanges cannot resolve the constituents and we can view them as nonoverlapping and even point like.

(iii) The coupling to other "inelastic channels" is not of critical importance.

The meson-antimeson bound states mentioned above are quite distinct from the $\bar{q}q$ bound states of the quark model which are generated by the confineme mechanism. The situation here may be similar to that of $\bar{N}N$ threshold states.[2,3] A formal distinction between states generated via confining and non-confining part of the potential in a two channel $\bar{q}q$-$\bar{M}M$ situation has been recently proposed.[4,5]

It would be desirable to start from a basic theory, like Q. C. D., where the original $\bar{q}q$ states are generated by the basic color confining mechanism which conceivably could be approximated in certain cases by linear potentials. The remaining "Van Der Waals" type forces between mesons would then be responsible for the molecular-like "exotic" threshold (i. e. $\bar{q}q\ \bar{q}q$) bound states. An interesting attempt along this line has been recently made by A. Duncan.[6] Various suggestions on the possible role of exotic $q\bar{c}\,\bar{q}c$ [7] (or specifically di-quark--anti-di-quark[8]) mesons in the structure in R have also been made lately though some of their motivations have disappeared with the recent discovery of low lying charmed particles.

In the absence of a really satisfactory first principle understanding of forces between two mesons, we will adopt in the following a pedestrian approach by assuming:

(a) Loosely bound $\bar{D}D$ (and conceivably $D^*\bar{D}$, $D^*\overline{D^*}$) states exist.

(b) The binding mechanism of these states is completely dominated by a few light meson exchanges. This tends to make the coherent sum of many exchanges of the dual like case (Fig. 2) inoperative. The $q\bar{q}\,q\bar{q}$ systems will then not fuse into a smaller $\bar{q}q$ system but will indeed be a $M\bar{M}$ loosely bound state.

We would like to reiterate that the crucial assumption is the clear distinction between these states and the quark model $\bar{q}q$ (say ψ, ψ', ψ'', etc. in the charmed case) states. While the two types of states will mix and jointly show up in specific channels such as $e^+e^- \to \bar{D}D$, $D^*\overline{D^*}$, $\overline{DD^*}$, etc. they will have quite distinguishable characteristics. We will not attempt in the following an exhaustive discussion of the coupled channel problem. The effects of mixing of the $D\bar{D}$ and $D^*\overline{D^*}$ components in the $c\bar{c}$ wave functions on radiative transitions and on the $\sigma_{Tot}(e^+e^- \to \text{hadrons})$ have already been considered[9] in some detail using two different approaches. In view of the recent experimental result on photonic transitions, it appears that there is no need for very strong mixing and distortion of the simple $c\bar{c}$ bound state picture.

II. THE $\bar{D}D$ CASE

A prime example of the states we have in mind is the deuteron and similar nuclear states. The exchanges of the lowest mesonic states π, ϵ, ρ, ω is sufficient for describing its binding (and the vast amount of information on NN scattering as well). Also the coupling to inelastic channels, e.g. N^*N^*, is presumably relatively small and does not affect the gross features of the bound state. It has been speculated that similar quasi-bound states can occur also in the $\bar{N}N$ system[3] and may correspond to the recently observed structure in $\bar{p}p$ annihilation. The analogy to $\bar{D}D$, $\overline{D^*}D^*$, $\bar{D}D^*$, etc. state is very suggestive.

The D's are (presumably) strongly interacting and can exchange ω's, ρ's, ϵ's, etc. with coupling roughly like those in the $\bar{K}K$ system. The ω and ρ couple in the conventional schemes only to the nonstrange quarks ($q = u$, or d) and the difference between K and D is only in the "spectator" heavy quark (s and c quark in the two cases respectively). We thus expect the SU(4) relations $g_{\bar{K}K\omega} = g_{\bar{D}D\omega}$ and $g_{\bar{K}K\rho} = g_{\bar{D}D\rho}$ to be (apart from weak "reduced-mass" effects) quite adequate. Alternatively $g_{\bar{K}K\rho}$ and $g_{\bar{D}D\rho}$ can be related via VDM to the same isovector charges of the D or K states and insofar as the isospin spatial distribution is similar in both cases the VDM hypothesis would have similar validity.

We make for the ensuing discussion the assumption that the light meson exchanges generate in the $\bar{K}K$ and $\bar{D}D$ systems a N. R. Yukawa potential $\frac{e^{-\mu r}}{r}$, $\mu = m_\epsilon = m_\omega = m_\rho = 600 - 800$ MeV with the same shape and somewhat reduced strength. Based on earlier estimates of $g^2_{\omega K\bar{K}}/4\pi$, $g^2_{\rho K\bar{K}}/4\pi$ and $g^2_{\epsilon K\bar{K}}/4\pi$, we expect $g^2_D/4\pi$ to lie in the range 2 to 8. The likelihood of generating a loosely bound threshold state in the $\bar{D}D$ system is then much larger as seen from the following simple scaling arguments.

$$\left\{\frac{-1}{m_D}\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{m_D r^2} + g_D^2 V(r)\right\} U_{D,i}^{\ell}(r) = E_{D_i}^{\ell}\, U_{D,i}^{\ell}(r) \tag{1a}$$

$$\left\{\frac{-1}{m_K}\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{m_K r^2} + g_K^2 V(r)\right\} U_{K,i}^{\ell}(r) = E_{K_i}^{\ell}\, U_{K,i}^{\ell}(r) \tag{1b}$$

are the radial equations for the ith state in the ℓth partial wave for the $\bar{D}D$ and $\bar{K}K$ systems respectively. If the second equation is satisfied for a given g_K^2 yielding some binding E_K then we readily see multiplying through by m_D (or m_K) that the first equation will yield an "Isomorphic" bound state with the same shape with a smaller binding $E_D = E_K\left(\frac{m_K}{m_D}\right) = \frac{E_K}{4}$ even if the coupling g_D^2 was ~ 4 times weaker than g_K^2. Thus it is much more likely to have, say, S and P-wave $\bar{D}D$ than $\bar{K}K$ threshold state and in particular it is conceivable that P-wave $\bar{D}D$ bound states exist and the corresponding $\bar{K}K$ states do not.

We note that for the potential model to be viable the binding should be indeed smaller than say $m_{D^*} - m_D = 150$ MeV so that like in the deuteron $D^*\bar{D}$ etc. admixtures will be small. The states with smaller bindings will also tend to have a slower asymptotic fall off $\sim e^{-\sqrt{M_D E_B}\, r}$ and hence more spatially extended wave function. It is really only in this case that the $D\bar{D}$ and $\bar{c}c$ states can be distinguished from each other. Another important feature is that the annihilation process $\bar{D}D \to$ ordinary mesons is presumably very slow. Diagrammatically such annihilations involve $D, D^*, \ldots$ exchanges which may be suppressed because of small Regge intercepts[10] if the Regge language is indeed applicable to threshold annihilations, and even more so because of the short range ($= \frac{1}{m_D}$) of the annihilation potential as compared to the size of the loosely bound system. This is similar to the effect in $\bar{N}N$ threshold states but perhaps even more pronounced here since the effective annihilation volume is reduced by $\left(\frac{M_D}{m_N}\right)^3 = 8$.

As a continuation of the earlier work on $\bar{N}N$ bound states, attempt is being made to do a more exhaustive analysis including all the SU(4) baryons.[11] However, the charmed baryon-anticharmed baryon states may be prone to annihilation into $\bar{D}D$, $\bar{D}DM$, $D^*\overline{D^*}$, etc. via noncharmed nonstrange baryon exchanges. No such lower mass charm-charm bound states exist for $D\bar{D}$ bound states to decay into which may make it even a better candidate for an ideal loose bound state in the sense described above.

II. ESTIMATE AND CALCULATION OF BOUND STATES

We now proceed to estimate the feasibility of S and P wave "pure" $\overline{D}D$ bound states using eq. (1a). First let us use general inequalities to assess the likelihood of binding. The Bargman[12] inequality

$$1 \leq n_\ell = \text{No. of bound states in angular momentum } \ell < \frac{1}{2\ell + 1}\int_0^\infty dr\, rM_D V(r)$$

is readily satisfied for $\ell = 1, 2$. For S wave the following Calogero inequality[12] $n_0 < \frac{2}{\pi}\int_0^\infty dr \sqrt{M_D V(r)}$ is slightly more restrictive for Yukawa potential. Yet the above range of parameters is consistent with several S wave bound states.

One would be more interested to see if conditions which are sufficient to insure the existence of L wave bound states can be met. Such a condition is Calogero's (Eq. 38, Chap. 23)[12]

$$\max_R \left\{ \int_0^R dr\, rM_D V(r)\left(\frac{r}{R}\right)^{2L+1} + \int_R^\infty dr\, rM_D V(r)\left(\frac{r}{R}\right)^{-(2L+1)} \right\} \geq 2L + 1 \quad .$$

Choosing $R = 1/\mu$ we find that the inequality can be met. Note that all the exchanges give rise to attraction in the I = 0 $\overline{D}D$ channel. For isovector states the ρ exchange becomes repulsive $\left\{ \tau_1 \cdot \tau_2 = \begin{matrix} -3 \text{ if } I = 0 \\ 1 \text{ if } I = 1 \end{matrix} \right\}$ and the binding is less likely. The effective coupling $g_D^2/4\pi$ for this case is closer to the lower limit mentioned above. See discussion preceding eq. 1(a).

Since no exact analytic solutions are available for the case of Yukawa potential, we solved the partial wave equation using a numerical code.[13]

This is clearly much more useful than using the general bounds which saturate for very special potentials. We fixed the mass of the effective exchange $m \approx m_\omega \approx m_\rho \approx m_\epsilon$ to be 750 MeV. The only parameter which needs to be varied is then the strength $g^2_{eff}/4\pi$ ranging between ≈ 10 (exact SU(4) and duality limit) and ≈ 2. The results are shown in Fig. 5. We note in particular that for $2 \leq g^2{}_{eff}/4\pi \leq 3.5$ the S-wave state has small binding ($0 \leq E_{1S} \leq 100$ MeV) and for $5.4 \leq g^2{}_{eff}/4\pi \leq 7$ the P-wave state has a small binding ($0 \leq E_{1P} \leq 150$ MeV). The root mean squared radii, $\sqrt{\langle r^2 \rangle}$, $\left(\langle r^2 \rangle = \int dr\, r^2 |U(r)|^2 / \int |U(r)|^2 dr\right)$ of some of these states are shown in the table I. The rms values should be contrasted with the rms of the ψ or the P-wave states in the charmonium model which are $\langle r^2 \rangle^{1/2}_{1S} = .28$ fm and $\langle r^2 \rangle^{1/2}_{1P} = 0.41$ fm.[14] It is precisely when the couplings fall in the above ranges and we have these large loosely bound states that the overlap with the conventional tightly bound $c\bar{c}$ states is small. We extended the above calculations to states with nodes like radially excited 2S-state. It is amusing to note that for Yukawa potential with the parameters in the above range, the correct level ordering i.e. $|E_B|_{1S} \geq |E_B|_{1P} \geq |E_B|_{2S}$ is obtained which is the same level ordering as the one obtained using confining potential. It is clear from Fig. 5 that the $\bar{D}D$ system will have a P-wave resonance if $g^2_{eff}/4\pi$ is ~ 5 which will give rise to a structure in $\sigma_{Tot}(e^+e^- \to \bar{D}D)$.

So far our discussion has been limited to the bound state and resonances in the $\bar{D}D$ system. A similar calculation of bound states and resonances in the $D^*\overline{D^*}$ and $D^*\bar{D}$ systems is in progress and the results will be published elsewhere.

III. EXPERIMENTAL CONSEQUENCES

The 0^+ S wave $\bar{D}D$ bound states will be very difficult to observe experimentally. In principle such bound states may occur in hadronic production in a small fraction of the associated charmed production events--much in the same way as say antideuterons are produced in high energy pp collisions. In practice the detection of such states may be very difficult particularly since they are not really absolutely stable because of the possibility of annihilations. The most deeply bound $\bar{D}D$ 0^+ states could in principle participate also in $\begin{array}{l}\psi' \to X + \gamma \\ \quad\hookrightarrow \psi + \gamma\end{array}$ cascades. In principle if one does not identify the fourth state which appears to participate in such cascades with the $\eta_c{}'$ because of several difficulties[15] there is certainly room for such additional states. The specific details of their effect require however estimates of radiative transitions $\begin{array}{l}\psi' \to (\bar{D}D) + \gamma \\ \quad\hookrightarrow \psi + \gamma\end{array}$ which are not attempted here. Clearly one needs to assess if an "exotic" 3.45 GeV state will indeed be very narrow as the Chanowitz and Gilman analysis[15] requires. An $I = 0$ $L = 0$ $(\bar{D}D)_{3.45}$ state cannot cascade-decay strongly into $\psi\pi^0$ or $\psi(\pi^+\pi^-)_{I=L=1}$ $\{$I-spin$\}$ nor into $\psi(\pi^+\pi^-)_{I=L=0}$ $\{$charge conjugation$\}$. The direct hadronic decays $(\bar{D}D_{3.45} \to$ hadrons will be <u>much</u> suppressed if as in the original suggestion of H. P. Durr[16] for the $\psi(=\Omega\bar{\Omega})$ the $\bar{D}D$ are indeed very far on the average and we have a very large state with a small magnitude of $\psi(0)$ the wave function at the origin.

The P wave 1^{--} $D\bar{D}$ bound states can be directly produced in e^+e^- collisions, and thus in principle have a much stronger experimental signature. The method that we suggest to ascertain their effects and in particular disentangle them from the effects of prominent pure $c\bar{c}$ bound states (which become sometimes resonances because of the availability of $\bar{D}D$ etc. decay channels) is based on the following simple 2 step picture of the (say $\bar{D}D$) production process Fig. 6. The photon first converts into the pointlike $c\bar{c}$ pair which have their own confining, say, gluon exchange and effectively linear potential and the ensuing $c\bar{c}$ ^3S bound states $\psi(3.1)$, $\psi'(3.7)$ ... At the second stage an additional $\bar{q}q$ quark pair is generated and by now we have the spatially more extended $\bar{D}D$, $D^*\bar{D}$, etc. states. In these states the longer range meson exchange interactions prevails yielding $\bar{D}D$ etc. bound states and resonances. If the space and time scales of the two assumed stages are sufficiently distinct we can apply the "two potentials" or the Watson final state interaction theorem. It would assert that the effect of the final state $D\bar{D}$, $D^*\bar{D}$, etc. interaction is simply to multiply the partial wave amplitude $T_{''\gamma''} \to (\bar{D}_i D_j)$ by the multichannel analog of the phase factor $e^{i\delta_{\text{final}}}$ namely the square root of the S-matrix: $<(\bar{D}_i D_j)|S^{\frac{1}{2}}|(\bar{D}_i D_j)>$. Since the S matrix and its square root are clearly unitary this final state phase is not going to affect the total $\sigma^{tot}_{''\gamma''} \to$ hadrons or more specifically $\sigma^{tot}_{''\gamma''} \to$ charmed hadron pairs which in the threshold region is proportional to $\sum_{ij} |T(''\gamma'' \to D_i\bar{D}_j)|^2$. Thus the only resonances which should be manifest in $\sigma_{tot}(e^+e^- \to$ charmed hadrons) should be the bona fide $c\bar{c}$ resonances. The effect of the $\bar{D}D$, $D^*\bar{D}$, etc. states will be manifest then only in the branching of that given primary total e^+e^- hadron cross section into the various available final $D_i\bar{D}_j$ channels. If a $D_i\bar{D}_j$, $D_i^*\bar{D}_j$ and $D_i^*\overline{D_j^*}$ resonance state exists close to the relevant $W = M_i + M_j$ threshold, then it will show up via an enhancement in that final state cross section (after taking away the obvious kinematic threshold factor).

Clearly all the above is an idealization and rests on the assumption that the time scale for the final state interactions T_{final} is very long so that the relevant structures due to it in $\sigma_t(\bar{D}D)(s)$ are very narrow. In practice we may need to average over an energy interval $\Delta W = \frac{1}{T_{final}}$ so as to ensure that the net contribution of a $\bar{D}_iD_j$ threshold state to σ_{tot} will indeed vanish. This is very similar to the suggestion of Ref. 5 and the earlier work of R. Dashen and G. Kane[4] that the net contribution of an "accidental" state--i.e., a state generated via the potential in the nonconfining channels--to finite energy sum rules or to the Levinson's theorem integral vanishes almost locally. In addition to the above $I = 0$ $D\bar{D}$ bound states and resonances this analysis indicates the possibility of an $I = 1$ bound state. Such a state will be distinct from and will not mix with any of the $c\bar{c}$ bound states. Therefore, based on this study we conclude that there exists the possibility of two charmed meson bound states and resonances i.e. "charm molecules", and, therefore, a new spectroscopy.

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Goldhaber, G. Kane, I. J. Muzinich, J. Rosner and T. L. Trueman for useful remarks.

After the completion of this work we learned about a preprint by L. B. Okun and M. B. Voloshin[17] which also discusses the possibility of $D\overline{D}$ bound states.

TABLE I

Numerical Values of $\sqrt{<r^2>}$ for specific $D\bar{D}$ Bound States

S-Wave (nodeless)			P-Wave (nodeless)		
S. T. = 2.40 B. E. = 14.82	= 3.0 = 53.10	= 4.0 = 160.87	= 5.33 = 8.33	= 6.0 = 69.54	= 7.0 = 204.60
1.11	0.72	0.51	0.94	0.58	0.45

r = fm
B. E. = Binding energy (MeV)
S. T. = Strength of the potential

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FIGURE CAPTIONS

Fig. 1: A schematic illustration of a bound state in meson anti-meson system in the quark model framework.

Fig. 2: Sum of t-channel exchanges in meson-meson scattering. These crossed channel exchanges provide the binding mechanism in the direct channel.

Fig. 3: Exchanges leading to excited $M^{*}M^{*}$ state.

Fig. 4: An illustration of the overlap of the two mesons and relevance of short range exchanges.

Fig. 5: Binding energies as function of the strength of the potential for 1S, 1P, and 2S $D\bar{D}$ bound states (nodeless).

Fig. 6: Two stage picture for the process $e^{+}e^{-} \rightarrow D\bar{D}$ and formations of the P-wave resonance while exchange of light mesons.

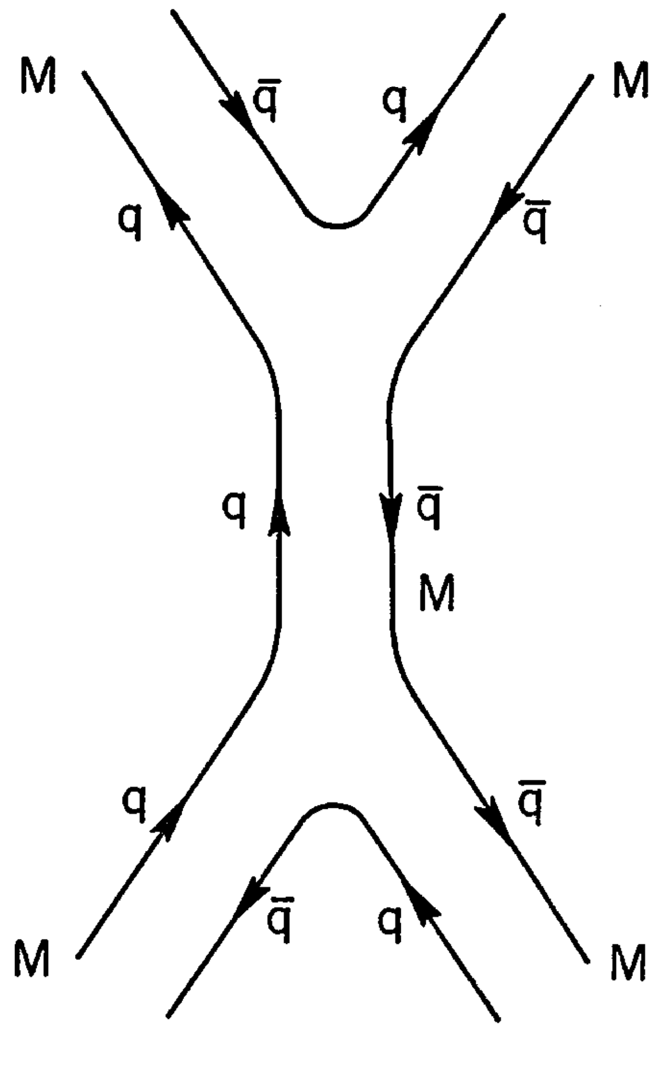


Fig. 1

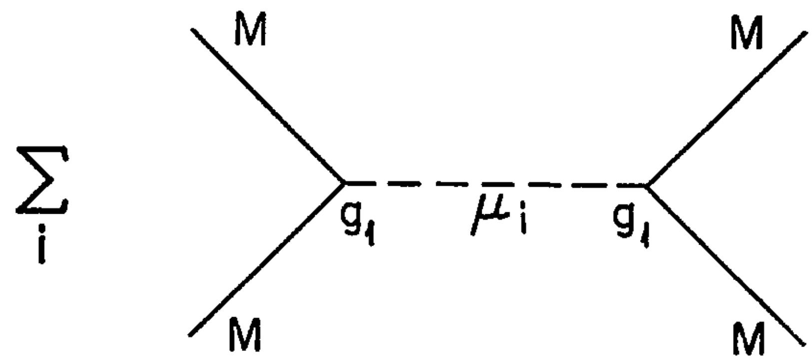


Fig. 2

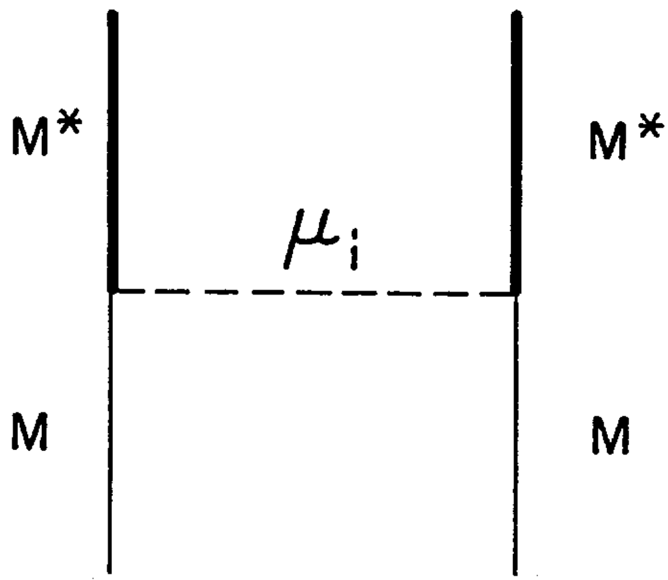


Fig. 3

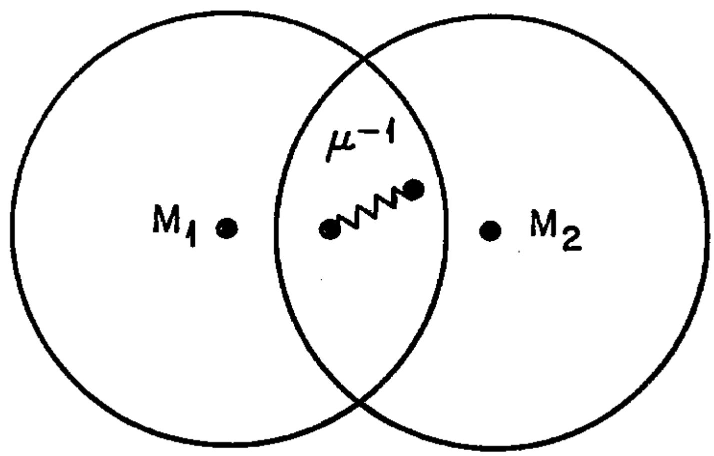
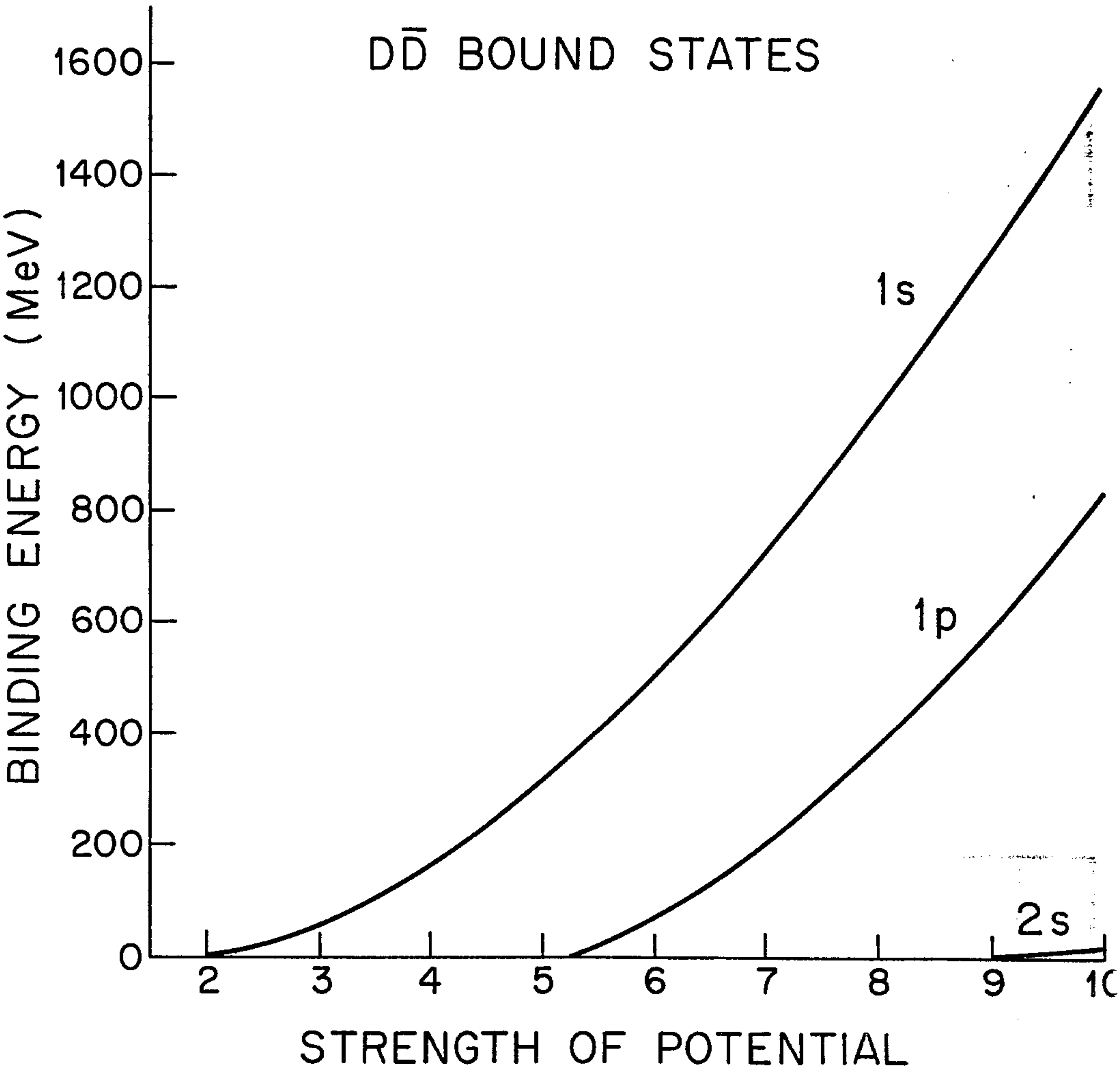


Fig. 4



$D\bar{D}$ BOUND STATES
BINDING ENERGY (MeV)
1600
1400
1200
1000
800
600
400
200
0
1s
1p
2s
2
3
4
5
6
7
8
9
10
STRENGTH OF POTENTIAL

Fig. 5

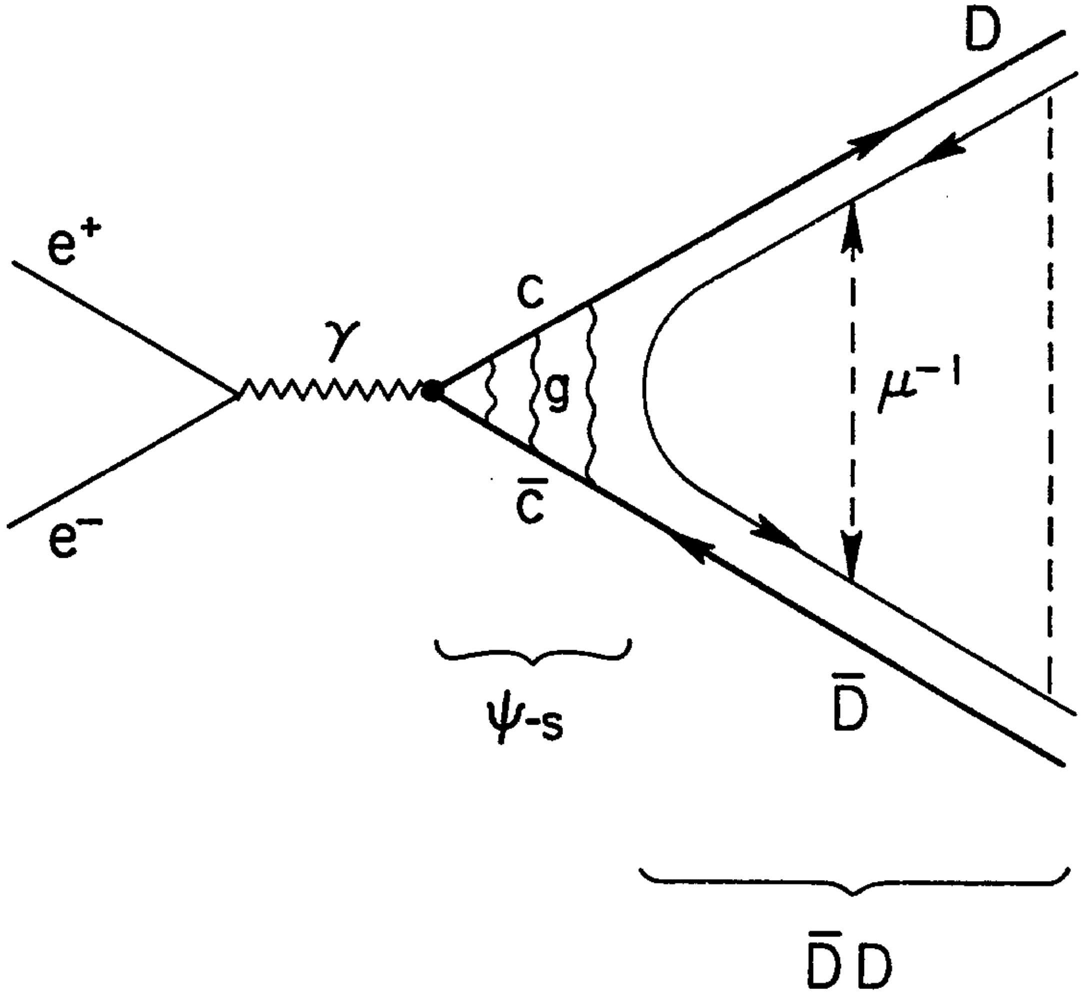


Fig. 6